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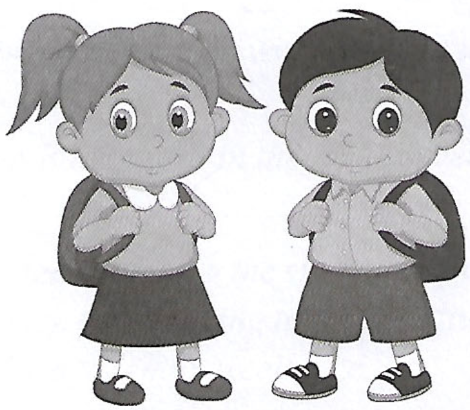
EVERSHINE

Learn and Grow

MATHEMATICS

Part-8



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Published by:
Evershine publishers

Designed and Illustrated by : B S Rawat (9015456544)

PREFACE

We are very pleased to present the new series 'Learn and Grow mathematics'.

The series comprises of eight books for classes I to VIII.

The series has been aimed at building a strong foundation with clear concepts and providing a great deal of practice in mathematics.

The Subject matter has been produced in such a way that it focuses on the development of the understanding, thinking and reasoning skill of the students.

The subject matter has been presented keeping in mind the principle that mathematics teaching involves the mastery of one skill before progressing to another. The age, the mental level and the difficulties faced by the students at all levels have also been thought of while presenting different concepts.

The latest syllabus prescribed by NCERT has been strictly followed by the New Series Learn and Grow.

Everything has been explained elaborately with plenty of illustrations so that the things might be crystal clear.

- *Lab Activity and Question Bank given in the end of every chapter.*
- *Model Test Papers are also given.*

No stone has been left unturned in making the students equipped with the ability to understand and solve problems confidently. Challenging tasks and situations have been created for bright students to motivate them for academic excellence.

Suggestions for the improvements of the book will be gratefully acknowledged.

Authors

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1 Rational Numbers

INTRODUCTION

In previous classes we have learnt about rational numbers. We have also studied the representation of rational numbers on a number line, comparison of rational numbers and four fundamental operations on rational numbers. Here, we shall learn about all these things in a detailed way. We shall also learn properties of rational numbers and solve word problems related with rational numbers.

RATIONAL NUMBERS

The numbers which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **rational numbers.**

EXAMPLE :

$\frac{2}{3}, \frac{4}{7}, \frac{-1}{3}, \frac{-4}{3}, \frac{7}{-9}, \frac{5}{1}, \frac{-11}{1}$, ... etc.

REMEMBER:

(*i*) All natural numbers, integers and fractions are rational numbers, but a rational number may or may not be a natural number or an integer or a fraction.

(*ii*) Zero is also a rational number as it can be representated in the form of $\frac{p}{q}$ as $\frac{0}{1}, \frac{0}{2}, \frac{0}{-4}$, etc.

POSITIVE AND NEGATIVE RATIONAL NUMBERS

If the numerator and denominator are either both positive integers or both negative integers, then the rational number is a **positive rational number.**

EXAMPLE :

$\frac{1}{3}, \frac{4}{7}, \frac{-5}{-11}, \frac{-9}{-14}$ ... etc.

If the numerator and denominator of a rational number are of opposite signs, then the rational number is a **negative rational number.**

EXAMPLE :

$\frac{-7}{8}, \frac{-1}{5}, \frac{3}{-4}, \frac{11}{-19}$ ... etc.

REMEMBER:

(*i*) Every natural number is a positive rational number.

(*ii*) The value of a positive rational number is greater than zero and the value of a negative rational number is less than zero.

ABSOLUTE VALUE OF A RATIONAL NUMBER

The absolute value of a rational number is its numerical value without considering its sign.

For representing the absolute value of any rational number we put a bar on the rational number.

EXAMPLE :

If a is any rational number, then the absolute value of a is written as $|a|$

Therefore, $\left|\frac{2}{3}\right| = \frac{2}{3}$, $\left|\frac{-3}{-5}\right| = \frac{3}{5}$ $\left|\frac{-4}{7}\right| = \frac{4}{7}$, $\left|\frac{7}{-9}\right| = \frac{7}{9}$

REMEMBER:

(*i*) The absolute value of every rational number is always equal or greater than the rational number.

(*ii*) The absolute value of every rational number except zero, is always positive.

(*iii*) The absolute value of zero is zero itself.

Thus, if 'a' is an integer then, $|a| = a$ if $a > 0$

$= 0$ if $a = 0$

$= a$ if $a < 0$

EQUIVALENT RATIONAL NUMBERS

Consider the following rational numbers.

(*i*) $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8} = \frac{6 \times 4}{8 \times 4} = \frac{24}{32}$

Here, $\frac{3}{4}, \frac{6}{8}, \frac{24}{32}$ are equivalent rational numbers.

(*ii*) $\frac{250}{325} \div \frac{5}{5} = \frac{50}{65} \div \frac{5}{5} = \frac{10}{13}$

Here, $\frac{250}{325}, \frac{50}{65}, \frac{10}{13}$ are all equivalent rational numbers.

NOTE :

❖ If $\frac{p}{q}$ is a rational number and n is a non-zero integer, then

(*i*) $\frac{p}{q} = \frac{p \times n}{q \times n}$ (*ii*) $\frac{p}{q} = \frac{p \div n}{q \div n}$

STANDARD FORM OF A RATIONAL NUMBER

Rational number is said to be in standard form if it is in its lowest term. To express a rational number in its lowest form:

(*i*) Make the denominator positive, if it is already not so.

(*ii*) Divide the numerator and denominator by their H.C. F.

EXAMPLE :

Express the following rational numbers in their standard form:

(*a*) $\frac{45}{70}$ (*b*) $\frac{-138}{92}$

SOLUTION :

(*a*) We have, $\frac{45}{70}$

H.C.F. of 45 nd 70 is 5

$$\frac{45}{70} = \frac{45 \div 5}{70 \div 5} = \frac{9}{14}$$

(*b*) We have, $\frac{-138}{92}$

H.C.F. of 138 nd 92 is 46.

$$\frac{-138}{92} = \frac{-138 \div 46}{92 \div 46} = \frac{-3}{2}.$$

NOTE :

- A rational number $\frac{p}{q}$ is said to be in standard form, if its denominator q is positive integer and p and q have no common factor other than 1.

COMPARISON OF RATIONAL NUMBERS

Method I. (*i*) Express each rational number with positive denominator.

(*ii*) Find the L.C.M. of the positive denominators.

(*iii*) Express each of the given rational numbers with their L.C.M. as the common denominator.

(*iv*) The rational number with greater numerator is greater.

EXAMPLE :

Compare $\frac{-7}{8}$ and $\frac{3}{-5}$.

SOLUTION :

$$\frac{-7}{8} = \frac{-7}{8} \text{ and } \frac{3}{-5} = \frac{-3}{5}$$

L.C.M. of 8 and 5 = 40

$$\therefore \quad \frac{-7}{8} = \frac{-7 \times 5}{8 \times 5} = \frac{-35}{40}$$

$$\text{And} \quad \frac{-3}{5} = \frac{-3 \times 8}{5 \times 8} = \frac{-24}{40}$$

Since, $-35 < -24$

$$\therefore \quad \frac{-35}{40} < \frac{-24}{40}$$

$$\Rightarrow \quad \frac{-7}{8} < \frac{-3}{5}$$

Method II : If a and c are integer and b and d are two positive denominators, then

$$\frac{a}{b} > \frac{c}{d}; \text{ if and only if } ad > bc.$$

$$\frac{a}{b} < \frac{c}{d}; \text{ if and only if } ad < bc. \qquad \left[\because \frac{a}{b} \times \frac{c}{d}\right]$$

$$\frac{a}{b} = \frac{c}{d}; \text{ if and only if } ad = bc.$$

REPRESENTATION OF RATIONAL NUMBERS ON A NUMBER LINE

In our earlier class, we have learnt about representation of natural numbers and integers on a number line. Here we shall learn representation of rational numbers on a number line.

When we have to represent positive and negative integers on a number line, we draw a straight line. Take a point O representing zero. Then we represent positive integers to the right and negative integers to the left of zero.

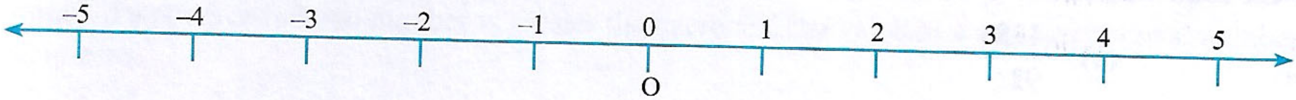

Fig. 1.1

Rational numbers are also represented on a number line in the same way. The positive rational numbers lie to the right of zero on the number line and negative rational numbers lie to the left of zero.

Suppose we have to represent $\frac{1}{3}$ on the number line then first we draw a line and mark a point 0 on it to represent rational number zero. Then we mark a point A on the line to the right of O. After that we divide OA in three equal parts OB = BC = CA. First part of segment OA represents rational number $\frac{1}{3}$. Thus, B represent rational number $\frac{1}{3}$.

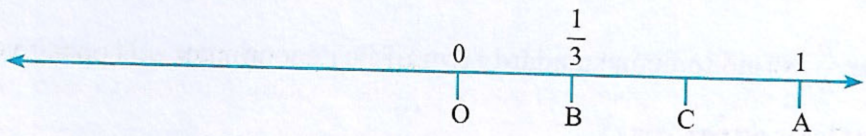

Fig. 1.2

Similarly, we can represent $\frac{-1}{3}$ on a number line by making point B′ on it. It will lie at the same distance as from O to B but in the opposite direction.

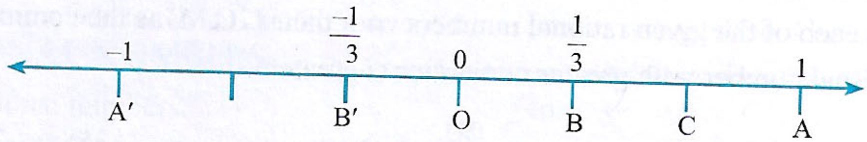

Fig. 1.3

EXAMPLES

EXAMPLE–1

Represent (*a*) $\frac{2}{5}$ (*b*) $\frac{-7}{3}$ on the number line.

SOLUTION :

(*a*) Draw a number line. Mark a point O to represent 0 and another point A to represent the distance 2 units. Divide OA into 5 equal parts (equal to the denominator of $\frac{2}{5}$), at P, Q, R and S. (Fig. 1.4.).

The point P represents the rational number $\frac{2}{5}$.

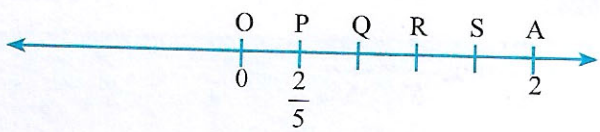

Fig. 1.4

(*b*) Draw a number line. Mark a point O to represent 0 and a point A′ at a distance of 7 units on the left of O to represent –7. Divide OA′ into 3 equal parts at P′ and Q′.

The point P′ represents $\frac{-7}{3}$ (Fig. 1.5).